# Numerical Integration
# Simpson's 3/8 Rule

# Objectives

- Numerically integrate a function using Simpson′s 3/8 Rule
- Simpson′s 3/8 rule falls under the closed form of the Newton-Cotes integration formulas.
- Newton-Cotes formulas are the most common integration schemes.
- Here we replace a complicated function or tabulated data using an approximating function that is easy to integrate.

# Newton – Cotes Formula

- $I = \int_a^b f(x)\,dx \sim = \int_a^b f_n(x)\,dx$
- $f_n(x)$ – polynomial of the form
- $f_n(x) = a_0 + a_1 * x + \ldots\ldots + a_{n-1} * x^{n-1} + a_n * x^n$
- n – order of the polynomial
- n = 1 in Fig 1 (1st order polynomial - Straight line)
- n = 2 in Fig 2 (2nd order polynomial - Parabola)
- n = 3 (3rd order polynomial)

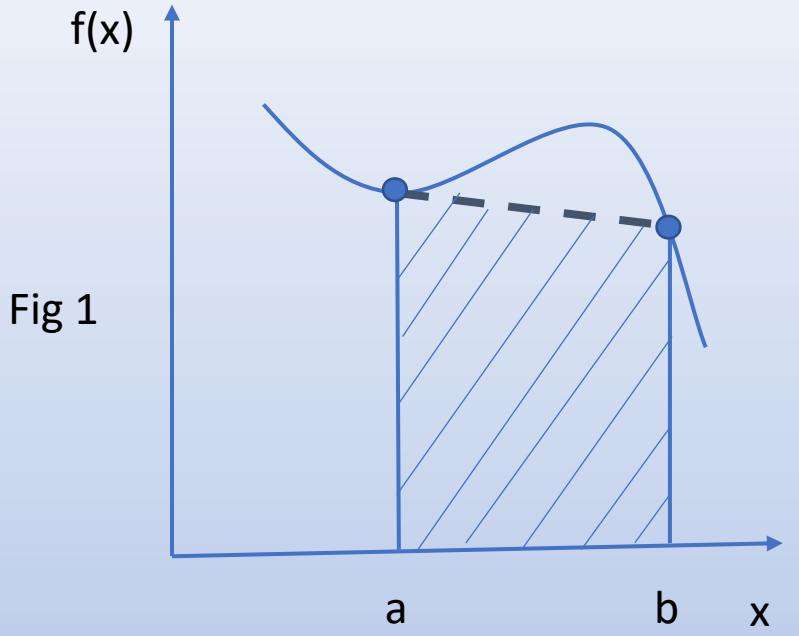


Fig 1

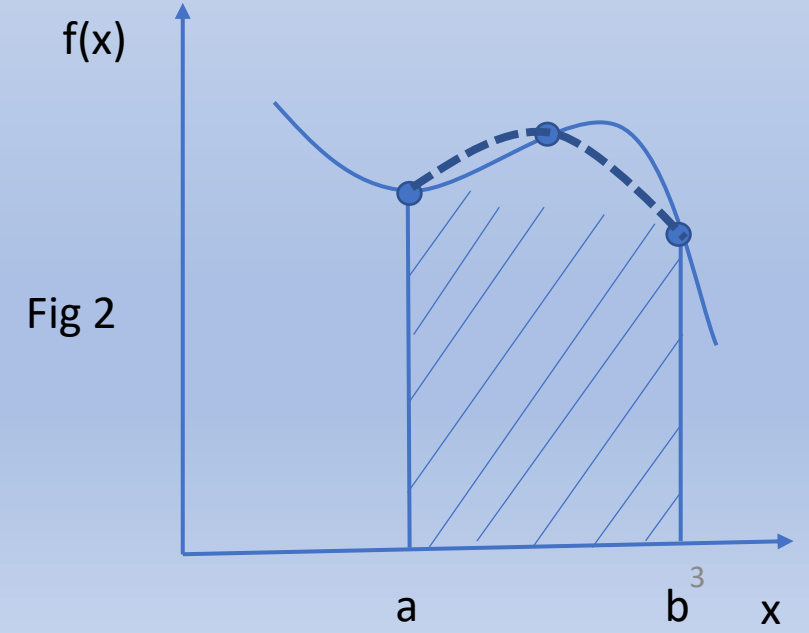


Fig 2

# Simpson's 3/8 Rule

- n = 3 (3rd order polynomial)
- Simpson's 3/8 rule is obtained by fitting a third-degree polynomial to four equally spaced discrete points
- $I = \int_a^b f(x)\,dx = \int_a^b f_3(x)\,dx$
- $f_3(x)$ is represented by a third-order Lagrange polynomial
- For a single interval of three increments:
- $I = (\frac{3}{8}) * h * (f_0 + 3 * f_1 + 3 * f_2 + f_3)$

# Simpson's 3/8 Rule

- The composite Simpson's 3/8 rule for equally spaced points is obtained by applying previous equation over the entire range of integration. Note that the total number of increments must be multiples of 3.

- $I = (\frac{3}{8}) * h * (f_0 + 3 * f_1 + 3 * f_2 + 2 * f_3 + 3 * f_4 + \ldots + 3 * f_{n\_1\_} + fn)$

- Global error – $O(h^4)$ ; h – interval size

# Simpson's 3/8 Rule

- Example – Integrate the function
- f(x) = 0.2 + 25*x - 200 * x^2 + 675*x^3 - 900*x^4 + 400*x^5;
- From a = 0 to b = 0.8
- The value obtained analytically is 1.640533
- Evaluate the integral by varying the no. of segments, n

# Summary

In this video,

- We presented Simpson's 3/8 Rule to numerically integrate a function f(x).
- The error is $O(h^4)$ (same as Simpson's 1/3 Rule) .
- Simpson's 3/8 rule can be used when the number of segments is odd.
- In some cases a combination of these 2 methods can be utilized. Three increments can be evaluated by the 3/8 rule, and the remaining even number of increments can be evaluated by the 1/3 rule.
- In the next video we can look at other methods such as Gauss Quadrature Methods.